

Deriving Open-Boundary Radiation-Pressure of the Electromagnetic Quantum-Flux

When Max Planck first modeled Black Body Radiation, he treated atoms as harmonic oscillators. After much effort, he finally matched experimental results with the following formula for the energy of a single photon..

1) Plancks Harmonic Oscillator Formula

$$E(\omega) = \hbar \omega (n + 1 / 2); \quad n = \{0,1,2,3 \dots (n + 1)\}$$

2) Therefore, according to Quantum Mechanics, the Zero Point Energy Field is defined in terms of harmonic oscillators. The photons which comprise this Field have a ground state, a zero-point energy that is defined as follows: The Zero-Point Energy of a photon is the case where n = 0. This Leaves a Zero-Point Energy per mode of a given frequency:

$$E_m(\omega) = \hbar \omega / 2$$

3) Modes of one frequency (ω , $\omega + d\omega$) per unit volume:

$$m(\omega) = (\omega^2 / \pi^2 c^3) d\omega$$

4) Energy (E) density of all modes \mathcal{M} of one frequency (ω) in a physically unbounded one cubic meter volume \mathcal{M}^3 of "empty" space, disregarding thermal energy:

$$E_m(\omega) * m(\omega) = E(\omega) / m^3 = \hbar \omega^3 / 2\pi^2 c^3$$

5) Integrating $E_m(\omega)$ over a range of frequencies (Ω) yields:

$$E_m(\Omega) / m^3 = (\hbar \omega^4 / 8\pi^2 c^3)$$

6) Frequency (ω) to wavelength (λ) conversion:

$$(\lambda \omega = c) \implies (\omega/c) = (1 / \lambda)$$

$$E(\Omega) / m^3 = E(\lambda) / m^3 = \hbar c / 8\pi^2 \lambda^4$$

7) Energy to momentum conversion:

$$E(\lambda) / c = P(\lambda)$$

8) Converting Momentum (P) and time (t) to force (F):

$$F = \Delta P / \Delta t$$

$$F(\lambda) = [E(\lambda) / c] / [1 / c]$$

$$F(\lambda) = E(\lambda) = \hbar c / 8\pi^2 \lambda^4 = 4.0 * 10^{-28} / \lambda^4 \text{ Pa}$$

9) Force to Pressure, A = Area:

$$A * F(\lambda) / m^2 = Pr(\lambda) = A * (4.0 * 10^{-28} / \lambda^4) P$$

10) Calculating Ideal Net Pressure

$$Pr_1(\lambda) - Pr_2(\lambda) = Pr_{Net}(\lambda)$$

11) Factoring-In Material Characteristics

$$F(\lambda) = Pr(\lambda) * (R_1 - R_2) * A$$

12) The Casimir Two Plate Formula Conversion Factor: 3.26

$$Pr(\lambda) * 3.26 = CF = \pi^2 \hbar / 240 \lambda^4$$

This factor of difference arises because Casimir Experiments alter

the Quantum-Flux but Transparent Quantum Radiometers alter the very-responses of different surface textures and materials to the *unaltered*, identical Quantum-Flux densities that is equal and opposite on both sides.

A very narrow band of the Soft X-Ray Spectrum will be targeted, perhaps 1.1 to 1.2 nm. Even if obtaining Total Reflections from very shallow angles only, we can realistically hope to obtain a net force of several hundred kPa. The following chart assumes that we are only able to use 0.5 percent of the total radiation pressure that is attributable to each 0.1 nm bandwidth.

We are only concerned with the few narrow bandwidths that most strongly interact with materials we are using. Such an approach *will* lead to underestimating the total pressures that are acting on our thruster; but we should rather under-promise and over-deliver. In practice, even if we were to do a summation of the additional pressures of ever larger wavelengths, additional increments of pressure rapidly approach zero due to the $1/\lambda^4$ term. When doing a summation of ever-smaller wavelengths, we rapidly reach a point where the wavelengths simply pass through matter and contribute little or no net force to the thruster.